Dynamic Analysis – Investigating the long-term behaviour of Economies

Jürgen Kremer
RheinAhrCampus Remagen

February 12, 2007

Contents

1 The Economic cycle **2**
1.0.1 Agents . . . 2
1.0.2 Markets . . . 3

2 Model excluding the state **4**
2.1 Households . . . 5
2.2 Firms . . . 5
2.3 Macroeconomic relations between households and firms . . . 5
2.4 Heterogeneous households . . . 7
2.5 Dynamic analysis . . . 7
2.5.1 Distribution of wages to household groups . . . 9
2.5.2 Modelling the development of consumption over time . . . 13
2.5.3 The division of consumption among household groups . . . 14
2.5.4 Determining interest payments and interest income per household group . . . 15

3 Implementation and evaluation of Dynamic Analysis **15**
3.1 Method . . . 15
3.2 Implementation and results . . . 17

4 Model including the State **26**

5 Summary and conclusion **26**

Bibliography **28**

Introduction

The standard procedure in macroeconomics is *comparative-static analysis*. Starting from a state of equilibrium, a change in a parameter of the model is observed, and the resulting new state of equilibrium is investigated. See Felderer/Homburg [1], Mankiw [2], Stiglitz [4] or Otto [3].

As a rule a state of equilibrium is defined as the intersection of two curves. A typical example involves a market in goods with a falling demand curve and a rising supply curve, where the ordinate of the graph displays the price and the abscissa the quantity of goods. The unique intersection point that results shows the equilibrium price and the equilibrium quantity. A change in a macroeconomic parameter results in a displacement of the demand and supply curves. We can then investigate the location of the new intersection point.

In contrast to comparative-static analysis, this paper presents a complementary procedure that we call *dynamic analysis*. In this framework the chronological development of a macroeconomic model over several time periods is simulated and analysed.

It turns out that, under standard assumptions, the economies that are modelled and investigated are unstable in the long term, with the interest rate in the capital market being the crucial causal factor. In contrast, economies that are stable over time can be modelled if no risk-free saving or credit interest is permitted.

These models not only explain a range of crucial observable macroeconomic effects, such as rising unemployment, falling wages and increasingly unequal wealth distribution; they also offer the basis for a solution. When the results are applied to real economies, the crucial recommendation is that political and economic structures should be changed so that unproductive saving interest and risk-free credit interest tend towards zero.

1 The Economic cycle

Below we present the standard macroeconomic model of an economic cycle in a closed economy. This model is the basis of the dynamic analysis proposed in this article, see Figure 1.

1.0.1 Agents

Macroeconomics distinguishes firms, households and the state as economic agents.

Households Households supply their labour power. They

- receive income,
- consume goods and services,
- pay taxes,
- develop savings.

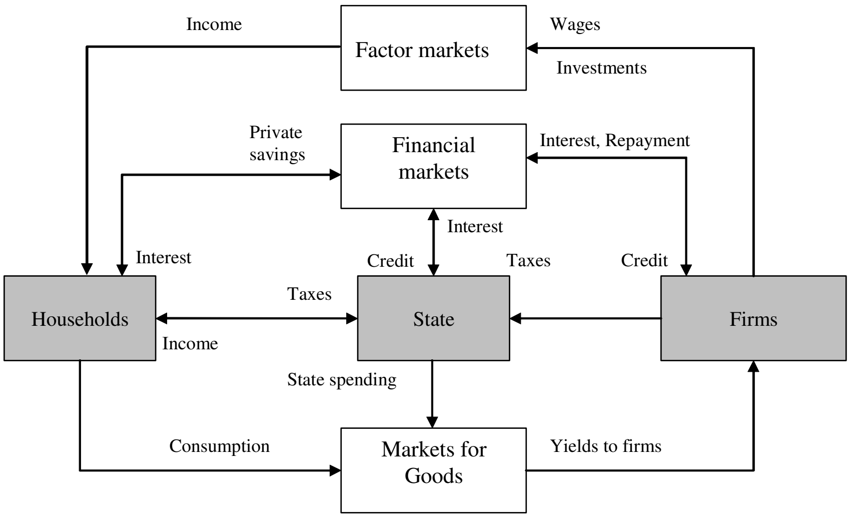

Figure 1: The Economic cycle

State The state employs workers. It

- pays wages,
- receives taxes,
- redistributes wealth through taxes and subsidies,
- invests in good and services,
- develops savings (especially negative savings, known as government debt).

Firms Firms produce goods and services and have a demand for workers. They

- receive income from the sale of goods and services,
- pay workers,
- invest (take up credits)
- pay taxes.

1.0.2 Markets

In macroeconomics we distinguish the following markets:

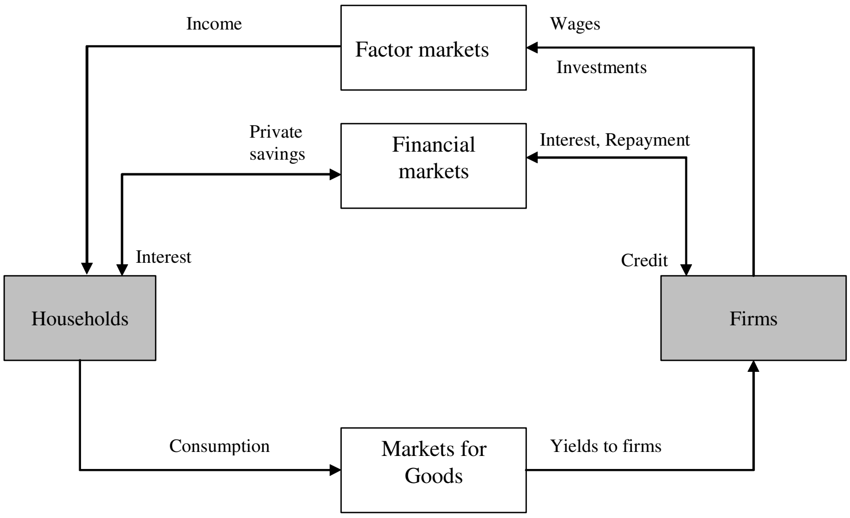

Figure 2: Economic cycle excluding the state

Factor markets The two most important production factors are capital and labour. Production factors is the term for inputs which are needed for the production of goods and services. Capital refers to all produced means of production which are used in the creation of goods. By labour we understand the time which the individual spends working for a wage.

Goods markets Here consumer and investment goods are supplied. Firms have a demand for investment goods, individuals for consumer goods. The state may demand either. Suppliers are firms.

Financial markets This is where the supply and demand for money is regulated. Typically this regulation is done by means of the interest rate.

2 Model excluding the state

We consider our economic model basically without taking into account the state as an agent. The involvement of the state is discussed in a separate article, but the economic role of the state in the dynamic analysis framework is briefly described qualitatively in section 4. The model used here is represented graphically in Figure 2. From the original diagram the agent State is now removed, along with all its links to other agents and markets. Henceforth we use the following abbreviations, fairly standard in economics:

Y	(Yield) GDP
P	(Price) Yields to firms
W	Wage
R	(Rate) Interest Rate
C	Consumption
I	Investment
T	Taxes
S	Savings
V	Value
h	household
f	firm

2.1 Households

Income from wages W and interest Rh from the saved wealth V of households are divided between consumption C and saving S. The amount saved is the difference between the incomes $W + Rh$ and expenditure C,

$$S = W + Rh - C. \tag{1}$$

If S is negative, then the saved wealth V of households is decreased by S. If S is positive, however, then V is increased by the amount of saving. If the entire wealth V is used up, then households start to become indebted. If household indebtedness is already present, then interest Rh is credit interest to be paid, and therefore negative.

2.2 Firms

Income of firms consists primarily of yield from sales P, while expenditure is comprised of wages W and interest costs Rf. For the difference, credit I must be taken on. We thus have

$$I = W + Rf - P. \tag{2}$$

If I is negative, it represents a repayment of previously received credit.

2.3 Macroeconomic relations between households and firms

Obviously it is still the case that

$$P = C. \tag{3}$$

The yield from sales by firms corresponds exactly to the expenditure on consumption by households. If saving happens in an economy, then a part of the wages W paid by firms is not used for consumption, and for firms the result is a need for credit of the amount saved S. Alternatively, there is investment in the economy. This means an increase in wages, which results in a higher rate of consumption or saving. From (1), (2) and (3) we obtain

$$\begin{array}{ccccccc} S & = & W & + & Rh & - & C \\ & & \shortparallel & & & & \shortparallel \\ I & = & W & + & Rf & - & P. \end{array} \tag{4}$$

Assuming that

$$Rh = Rf =: R \tag{5}$$

it follows from (4) that

$$S = I, \tag{6}$$

and we obtain the macroeconomic rule: *Savings = Investments.* In particular (5) follows in the special case $Rh = Rf = 0$.

As a consequence from (6) the aggregated saving V in an economy is exactly equal to the aggregated debts.

The correlation $S = I$ also means – following from (4) and (5) – that firms pay interest Rh as credit interest Rh on investment credit I. *The payment of interest on saving capital, and the increase in saving that this causes, thus impose increasing indebtedness and corresponding interest payments on firms.*

Henceforth we assume throughout that $Rh = Rf =: R$.

The **gross domestic product** or **gross national product** Y is defined as the sum of all yield by firms or expenditure by firms, that is, with (2) we derive

$$Y := P + I = W + R. \tag{7}$$

The reduction

$$Y = \frac{R}{Y}Y + \frac{W}{Y}Y \tag{8}$$

characterises the division of the Gross domestic product into an interest part $\frac{R}{Y}$ and a wages part $\frac{W}{Y}$. If interest costs R rise faster than gross domestic product Y, then the interest part $\frac{R}{Y}$ of gross domestic product increases and the wages part $\frac{W}{Y}$ decreases. *If gross domestic product rises less strongly than the interest part, then wage income in the economy decreases in accordance with (8). In this case an increasing part of the gross national product flows via interest payments to owners of capital in the economy, who in our model so far are identical with households.*

If, however, economic growth can be achieved, that is, rising gross domestic product Y, then the interest part $\frac{R}{Y}$ can be limited and the wages part $\frac{W}{Y}$ can be maintained, even though the interest burden R increases in absolute terms. *Thus limiting the interest part of gross domestic product necessitates constant economic growth.*

From (1) and (7) we also have:

$$Y = C + S. \tag{9}$$

2.4 Heterogeneous households

Hitherto we have not specified households more precisely but treated them as a homogeneous group. In fact, income and wealth conditions in real economies are very heterogeneous. Every household group receives interest from investing its savings. On the other hand, firms pass on their interest costs in the form of higher prices, so that every household group pays interest via consumption. *Groups with relatively low wealth have little or no income from interest, but pay so much interest via their consumption that they are net interest payers. For groups of relatively wealthy households, in contrast, income from interest outweighs interest payments. This group consists of net interest receivers.*

We model N classes of households which differ in their wealth and income. To do this we think of the economy as divided into N groups of equal numbers of people with equal value, income and consumption. Let

$$W = \sum_{i=1}^{N} W^i,$$

where W^i is the income of the i-th group, $i = 1, \ldots, N$. Correspondingly we have

$$C = \sum_{i=1}^{N} C^i,$$

and so on.

2.5 Dynamic analysis

We consider now the annual development of an economy over a period from today, $t = 0$, until T years in the future. The relationships presented in the previous sections hold for every year $t = 0, \ldots, T$. If we characterise all the quantities that appear through a time index t, we obtain:

Households	Balance equation	$S_t = W_t + R_t - C_t$
	Value	$V_t = V_{t-1} + S_t$
	Interest	$R_t = r_t V_{t-1}$
Firms	Balance equation	$I_t = W_t + R_t - P_t$
	Prerequisite for $S = I$	$Rf_t = Rh_t =: R_t$
	Gross domestic product	$Y_t = P_t + I_t = C_t + S_t = W_t + R_t$

We model economic growth with growth factors y_t for the time points $t = 1, \ldots, T$ by

$$Y_t = (1 + y_t)\, Y_{t-1}.$$

For gross domestic product we have the representations

$$Y_t = P_t + I_t = C_t + S_t = W_t + R_t.$$

If we define the **interest-excluded economic growth** rate y^w as

$$(1 + y_t^w)\, Y_{t-1} = W_t = Y_t - R_t,$$

it follows that

$$\begin{aligned} y_t^w \quad : \quad &= \frac{W_t}{Y_{t-1}} - 1 \\ &= \frac{Y_t}{Y_{t-1}} - \frac{R_t}{Y_{t-1}} - 1 \\ &= (1 + y_t) - \frac{R_t}{Y_{t-1}} - 1 \\ &= y_t - \frac{R_t}{Y_{t-1}}. \end{aligned}$$

The **interest-determined rate of price increase** y^r is defined by

$$(1 + y_t^r)\, W_t = Y_t = W_t + R_t.$$

It follows that

$$y_t^r = \frac{R_t}{W_t}.$$

Overall, then, we obtain

$$\begin{aligned} (1 + y_t)\, Y_{t-1} &= Y_t \\ &= (1 + y_t^r)\, W_t \\ &= (1 + y_t^r)\,(1 + y_t^w)\, Y_{t-1}. \end{aligned}$$

For the case where $R_t = 0$, we obviously have

$$y_t = y_t^w.$$

If we characterise the saved wealth up to time point $t - 1$ as V_{t-1}, then for interest R_t up to time point t we have

$$R_t = r_t V_{t-1}.$$

Here r_t denotes the annual interest between the time points $t - 1$ and t. From this we derive the total income W_t of the economy as

$$\begin{aligned} W_t &= Y_t - R_t \qquad (10) \\ &= Y_t - r_t V_{t-1}. \end{aligned}$$

We assume that for every household group $i = 1, \ldots, N$ at time point $t = -1$, initial wealth V_{-1}^i is available. From (10) it follows that wages W_t must fall if economic growth decreases while interest yield to households – and thus interest costs to firms – rise.

The total income W_t of the economy can now be distributed to the individual household groups. For this we define for $t = 1, \ldots, T$ factors $\alpha_t^i \geq 0$ with $\sum_{i=1}^N \alpha_t^i = 1$ and

$$W_t^i := \alpha_t^i W_t.$$

For $t = 0$ we assume an initial distribution $W_0^1, \ldots, W_0^N$ of incomes, and for each time point $t = 0, \ldots, T$ we have

$$\sum_{i=1}^N W_t^i = W_t.$$

We further assume that overall consumption changes with certain growth rates c_t, $t = 1, \ldots, T$, so that we have

$$C_t = (1 + c_t)\, C_{t-1}.$$

Further, individual household groups have to contribute to overall consumption C_t for $t = 1, \ldots, T$. For this we define, corresponding to wages, factors $\beta_t^i > 0$ with $\sum_{i=1}^N \beta_t^i = 1$ and

$$C_t^i := \beta_t^i C_t.$$

For $t = 0$ an initial division $C_0^1, \ldots, C_0^N$ is presupposed. In every case it follows that

$$\sum_{i=1}^N C_t^i = C_t.$$

Furthermore the amount of saving per household group is calculated:

$$\begin{aligned} S_t^i &= W_t^i + R_t^i - C_t^i \\ &= W_t^i + r_t V_{t-1}^i - C_t^i. \end{aligned}$$

Finally the amounts of wealth at time point t work out as

$$V_t^i = V_{t-1}^i + S_t^i$$

This procedure is now iterated over time. The simulation is terminated when either the final time point T is reached or when the overall income in the economy $W_t = 0$. In the latter case the share of interest in the gross domestic product reaches 100%.

2.5.1 Distribution of wages to household groups

For the distribution of wages W_t to the wages of household groups,

$$W_t = \sum_{i=1}^N W_t^i,$$

we present several variations in the model.

Wages are oriented to the gross domestic product of a group Here we assume that the wages of the i-th group are distributed in proportion to the share of that group in the gross domestic product. With

$$Y_t^i := W_t^i + R_t^i$$

we define, for $t = 1, \ldots, T$,

$$\begin{aligned} W_t^i &:= \alpha_t^i W_t, \\ \alpha_t^i &:= \frac{Y_{t-1}^i}{Y_{t-1}} =: \gamma_{t-1}^i, \end{aligned}$$

where

$$\gamma_t^i := \frac{Y_t^i}{Y_t}. \tag{11}$$

For $t = 0$ the wages for each group are given in advance, so we have

$$\alpha_0^i := \frac{W_0^i}{W_0}.$$

Each group's share of wages is constant Here we presuppose figures $\alpha^i \geq 0$ with $\sum_{i=1}^N \alpha^i = 1$, so that we have

$$W_t^i = \alpha^i W_t$$

for all values of t. The α^i in this case are taken from the initial distribution of incomes. We therefore have

$$\alpha^i := \frac{W_0^i}{W_0}.$$

Wages are oriented to the household groups in the economy with the highest incomes In the model this means that the wages of the group of households with the highest income is orientated to the growth of gross domestic product. The incomes of the remaining groups are adapted so that overall wages are consistent with (10), that is, the condition

$$W_t = Y_t - R_t$$

is fulfilled. If for interest payments $R_t = 0$, then by (10) it follows that $Y_t = W_t$, and increases in income correspond exactly to economic growth. If, on the other hand, $R_t > 0$, then incomes and their increases are less than the amount of, and the increase in, the gross domestic product. In this case, incomes cannot rise at an equal rate for all wage groups.

Not only is this model basically simple, it is also maximally realistic. The high wage groups participate fully in economic growth, while the income of the lower wage groups has to make do with what is left over.

First we define the projected incomes

$$\tilde{W}_t^i := (1 + y_t)\, W_{t-1}^i$$

for $i = 1, \ldots, N$. We assume that household groups are numbered in ascending order by income, so that the N-th group has the highest income. As the simulation is terminated when $W_t \leq 0$, we can always assume that $W_t > 0$. We consider the following different cases:

1st Case Under the assumptions

$$0 < \tilde{W}_t^N < W_t$$

and

$$\sum_{i=1}^N \tilde{W}_t^i \leq W_t$$

we define

$$\alpha_t^i := \frac{\tilde{W}_t^i}{\sum_{i=1}^N \tilde{W}_t^i} = \frac{W_{t-1}^i}{\sum_{i=1}^N W_{t-1}^i} = \alpha_{t-1}^i.$$

In this case the available total wage W_t exceeds the projected total wage $\sum_{i=1}^N \tilde{W}_t^i$, so that W_t can be distributed proportionally to the projected wages $\tilde{W}_t^i$ of each household group.

2nd Case As with the 1st case we assume

$$0 < \tilde{W}_t^N < W_t.$$

But now we consider the situation

$$\sum_{i=1}^N \tilde{W}_t^i > W_t.$$

Here the projected total wage $\sum_{i=1}^N \tilde{W}_t^i$ exceeds the available amount W_t. To reduce the wages of the lower groups $i < N$ to an amount below their projected values $\tilde{W}_t^i$ we look for a parameter $\lambda > 0$, so that

$$\begin{aligned} W_t &= e^{-\lambda(N-1)}\tilde{W}_t^1 + \cdots + e^{-\lambda}\tilde{W}_t^{N-1} + \tilde{W}_t^N \\ &= e^{-\lambda N} \sum_{i=1}^N e^{\lambda i} \tilde{W}_t^i. \end{aligned}$$

The greater factor λ is, the more strongly incomes for wage groups $< N$ decrease. With the following proof we demonstrate both the existence and the uniqueness of this parameter λ.

Theorem 1 *Assume values* $0 \leq \tilde{W}_t^1 \leq \cdots \leq \tilde{W}_t^N$ *and* W_t *with the properties*

$$0 < \tilde{W}_t^N < W_t \tag{12}$$

and

$$\sum_{i=1}^{N} \tilde{W}_t^i > W_t. \tag{13}$$

Then there is a uniquely determined parameter λ *such that*

$$W_t = e^{-\lambda N} \sum_{i=1}^{N} e^{\lambda i} \tilde{W}_t^i.$$

Proof. With $x := e^{-\lambda}$ we define a polynomial P as

$$P(x) := x^{N-1}\tilde{W}_t^1 + \cdots + x\tilde{W}_t^{N-1} + \left(\tilde{W}_t^N - W_t\right).$$

For $x = 0$ we have

$$P(0) = \tilde{W}_t^N - W_t < 0,$$

and for $\tilde{x} := \frac{W_t - \tilde{W}_t^N}{\tilde{W}_t^{N-1}} > 0$ we obtain

$$P(\tilde{x}) = x^{N-1}\tilde{W}_t^1 + \cdots + x^2\tilde{W}_t^{N-2} \geq 0.$$

Thus, by the intermediate value theorem, P has as a continuous function a root $x_0 \in (0, \tilde{x}]$. From (12) and (13) we conclude $\tilde{W}_t^{N-1} > 0$. Therefore we have

$$P'(x) = (N-1)\,x^{N-2}\tilde{W}_t^1 + \cdots + 2x\tilde{W}_t^{N-2} + \tilde{W}_t^{N-1} > 0$$

for all $x \geq 0$, and P is strictly increasing. Thus the root x_0 is uniquely determined. Finally we determine λ_0 by the condition $x_0 = e^{-\lambda_0}$, and the theorem is proved. ■

The root of P can be calculated by a standard method, such as the Newton algorithm. A natural initial value is

$$x := \frac{W_t - \tilde{W}_t^N}{\tilde{W}_t^{N-1}} = \frac{\frac{1}{(1+y_t)} W_t - W_t^N}{W_t^{N-1}}.$$

If the root $x_0 = e^{-\lambda_0}$ is found, then we have

$$\begin{aligned} W_t &= \sum_{i=1}^{N} e^{-\lambda_0 (N-i)} \tilde{W}_t^i \\ &= \sum_{i=1}^{N} \left(e^{-\lambda_0 (N-i)} \left(1 + y_t\right) \frac{W_{t-1}^i}{W_t} \right) W_t. \end{aligned}$$

Thus if we define

$$\begin{aligned} \alpha_t^i &:= e^{-\lambda_0(N-i)}\,(1+y_t)\,\frac{W_{t-1}^i}{W_t} \\ &= e^{-\lambda_0(N-i)}\,\frac{\tilde{W}_t^i}{W_t} \end{aligned}$$

and

$$W_t^i := \alpha_t^i W_t,$$

we obtain the correspondence

$$W_t = \sum_{i=1}^{N} \alpha_t^i W_t = \sum_{i=1}^{N} W_t^i.$$

The α_t^i thereby define the desired distribution of overall income W_t of the economy to individual household groups.

3rd Case

$$0 < W_t \leq \tilde{W}_t^N.$$

Here the income W_t to be divided is less than the amount $\tilde{W}_t^N$ which is to be allocated to the group with the highest income on the basis of economic growth. In this case we define

$$W_t^N := W_t$$

and

$$W_t^1 := \cdots := W_t^{N-1} := 0.$$

This therefore means that

$$\alpha_t^N = 1$$

and

$$\alpha_t^1 = \cdots = \alpha_t^{N-1} = 0.$$

In this case, then, we allocate the entire available income W_t to the highest income group.

2.5.2 Modelling the development of consumption over time

The starting point is always an initial consumption $C_0 = \sum_{i=1}^{N} C_0^i$.

The growth in consumption is oriented to the gross national product
We assume that consumption has the same growth as the overall economy. With $c_t = y_t$ we model

$$C_t = (1+y_t)\,C_{t-1}.$$

The growth in consumption is oriented to income Here we consider the share of wages in gross national product

$$\begin{aligned} w_t &: = \frac{W_t}{Y_t} \\ &= \frac{Y_t - R_t}{Y_t} \\ &= 1 - \frac{r_t V_{t-1}}{Y_t}, \end{aligned}$$

and we assume that consumption develops in line with this share. We thus model

$$C_t = (1 + c_t)\, C_{t-1}$$

with

$$c_t := \frac{w_t}{w_{t-1}} - 1.$$

Consumption stays constant over time A third possibility is to presuppose that consumption C_t stays approximately constant over time, so that $C_t = C_0$. If prices rise, households reduce their expenditure, and if prices fall, they increase their consumption. In this case we thus assume that the two effects approximately compensate for each other. In this case $c_t = 0$ for all $t = 1, \ldots, T$.

2.5.3 The division of consumption among household groups

Consumption is oriented to the gross domestic product of household groups One possibility is to orientate the consumption of a group to the gross domestic product of that group. Here we define for $t = 1, \ldots, T$ the consumption C_t^i of the i-th group by

$$\begin{aligned} C_t^i &: = \beta_t^i C_t, \\ \beta_t^i &: = \frac{Y_{t-1}^i}{Y_{t-1}} = \gamma_{t-1}^i, \end{aligned}$$

where (11) was applied. For $t = 0$ consumption is given in advance, so we have

$$\beta_0^i = \frac{C_0^i}{C_0}.$$

Each group's share of consumption is constant Here we presuppose figures $\beta^i > 0$ with $\sum_{i=1}^{N} \beta^i = 1$, so that

$$C_t^i = \beta^i C_t$$

for all $t = 0, \ldots, T$. In this case we have

$$\beta^i := \frac{C_0^i}{C_0}$$

2.5.4 Determining interest payments and interest income per household group

If we divide interest into percentages of consumption among households, we obtain as the interest payment hidden in consumption ρ_t^i of the i-th household

$$\rho_t^i := \frac{C_t^i}{C_t} R_t.$$

The balance of interest income R_t^i and interest payments ρ_t^i of the i-th household is:

$$R_t^i - \rho_t^i = r_t V_{t-1}^i - \frac{C_t^i}{C_t} R_t. \tag{14}$$

Clearly we have

$$\begin{aligned} \sum_{i=1}^{N} \left(R_t^i - \rho_t^i\right) &= r_t \sum_{i=1}^{N} V_{t-1}^i - \frac{R_t}{C_t} \sum_{i=1}^{N} C_t^i \\ &= r_t V_{t-1} - R_t \\ &= 0. \end{aligned}$$

3 Implementation and evaluation of Dynamic Analysis

First we present the theoretical concept of the simulation model. Then the implemented simulation program is briefly presented, and some notable results are developed and discussed.

3.1 Method

We assume that for $i = 1, \ldots, N$ the initial data

$$W_0^i, C_0^i \text{ und } V_{-1}^i$$

for income, consumption and wealth are given. In this study we model an interest rate r that is constant over time, but time-dependent economic growth factors y_t for $t = 1, \ldots, T$. Finally we presuppose that growth c_t of consumption corresponds to economic growth y_t, so that we have

$$c_t = y_t$$

for all $t = 1, \ldots, T$.

Initialising for $t = 0$. First we calculate

$$\begin{aligned}
Y_0^i &:= W_0^i + R_0^i, \\
R_0^i &:= rV_{-1}^i, \\
W_0 &:= \sum_{i=1}^{N} W_0^i, \\
R_0 &:= \sum_{i=1}^{N} R_0^i, \\
V_{-1} &:= \sum_{i=1}^{N} V_{-1}^i, \\
Y_0 &:= W_0 + R_0.
\end{aligned}$$

Thus we have

$$R_0 = rV_{-1}.$$

We define

$$\begin{aligned}
\alpha_0^i &:= \frac{W_0^i}{W_0}, \\
\beta_0^i &:= \frac{C_0^i}{C_0}.
\end{aligned}$$

Further, we obtain

$$S_0^i := W_0^i + R_0^i - C_0^i = Y_0^i - C_0^i$$

and

$$\begin{aligned}
V_0^i &:= V_{-1}^i + S_0^i, \\
V_0 &:= \sum_{i=1}^{N} V_0^i.
\end{aligned}$$

This concludes the calculations for the time point $t = 0$.

Calculations for the time point $t = 1$. First we have

$$\begin{aligned}
Y_1 &:= (1 + y_1)\, Y_0, \\
C_1 &:= (1 + y_1)\, C_0, \\
R_1 &:= rV_0.
\end{aligned}$$

We can now calculate

$$W_1 := Y_1 - R_1.$$

After determining the factors α_1^i and β_1^i for each household group we divide the overall income W_1 and the overall consumption C_1 among the individual groups:

$$W_1^i := \quad \alpha_1^i W_1,$$
$$C_1^i := \quad \beta_1^i C_1.$$

The interest yield for each group is

$$R_1^i := rV_0^i.$$

Thus we have

$$R_1 := rV_0 = \sum_{i=1}^{N} R_1^i.$$

For the gross domestic product of each group we thus obtain

$$Y_1^i := W_1^i + R_1^i.$$

We have

$$Y_1 = \sum_{i=1}^{N} Y_1^i.$$

From this the savings of households comes out as

$$S_1^i := W_1^i + R_1^i - C_1^i = Y_1^i - C_1^i,$$

And the wealth of individual groups at time point $t = 1$ is

$$V_1^i := \quad V_0^i + S_1^i,$$
$$V_1 := \quad \sum_{i=1}^{N} V_1^i.$$

This concludes the calculations for the time point $t = 1$. The procedure is now iterated for the further times $t = 2, \ldots, T$.

3.2 Implementation and results

To carry out the simulation the Java application *Dynamic Analysis* was developed. Currently it is also implemented as an Excel-VBA-application and is further freely available on the author's home page *www.rheinahrcampus.de/~kremer* along with all source code. The program is designed to verify empirically the theoretical statements of the study.

The user interface of the application *Dynamic Analysis* consists of the window illustrated in Figure 3. The left section displays the results of the simulation in a series of tables, which can be selected using tabs at the top. The different rows show the results of calculating from their initial values various macroeconomic quantities, such as wealth, income, consumption, saving, interest transfers and others over a 50 year time period.

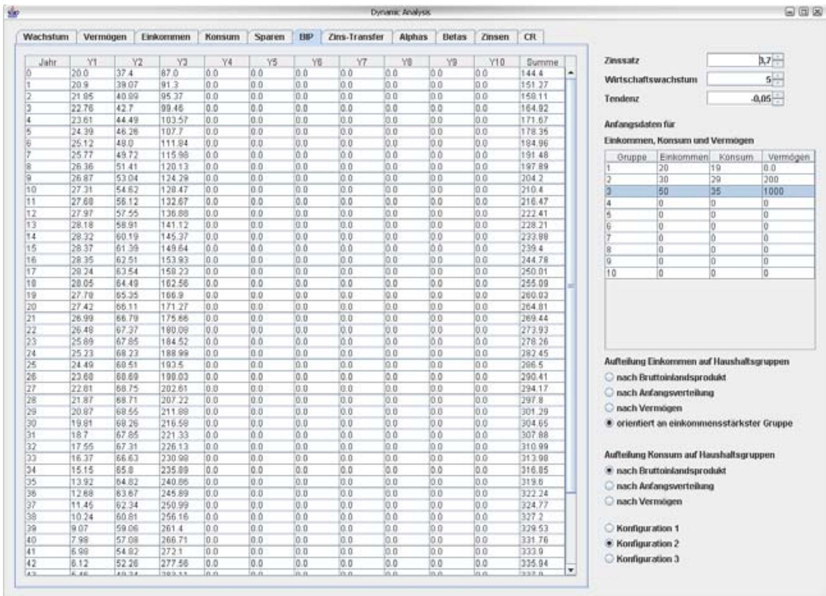

Figure 3: Java Application *Dynamic Analysis*

On the right the initial data for the simulation – interest rate and economic growth, along with the initial data for income, consumption and wealth for each household group – are entered. It is possible to simulate the development of an economy with up to ten household groups simultaneously.

For ease of reading the results of different simulations are shown here with only three household groups. The results are qualitatively the same, though more striking, when ten groups are considered.

In the following tables all monetary quantities can be interpreted as annual values in units of a *thousand euro*. The entries in columns r and y of the growth group are given in percentages, and the overall growth Σ is a dimensionless factor.

Initial data We stipulate for each of the three household groups G1, G2 and G3 the following initial data for income, consumption and wealth:

Initial data			
Group	**Income**	**Consumption**	**Wealth**
G1	20	19	0
G2	30	29	300
G3	50	35	1000

Dynamic Analysis For these initial data we consider the development of the economy under various assumptions for interest rate and economic growth. The time horizon is 50 years. For ease of reading, only the results for years $0, 10, \ldots, 50$ are given. The model calculates the distribution of income to individual households using the procedure which is oriented to the group with the highest income.

In distributing income we should note that there are limits on the extent to which consumption expenditure can be reduced. Food, rent, power and heating costs have to be paid as well as the cost of children's education. If prices rise, a reduction in consumption does not necessarily lead to a reduction in expenditure on consumption. Hence consumption expenditure was distributed to household groups within the analysis in proportion to the initial distribution.

In the tables on the following pages we abbreviate *Interest in consumption* to *Int. consumption*.

1. Scenario: Interest rate constant 3%, economic growth constant at 5%. The economy has constant interest rates and constant, strong economic growth.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	3	5	1.05	1	310	1045	20	39	80
10	3	5	1.71	20	437	1638	34	63	130
20	3	5	2.79	65	633	2602	57	101	212
30	3	5	4.54	158	936	4169	94	162	344
40	3	5	7.39	334	1409	6716	157	262	560
50	3	5	12.04	656	2151	10859	259	423	912

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1	10	45
10	33	50	83	31	47	57	3	15	73
20	55	82	137	50	77	93	6	24	119
30	90	135	225	82	125	151	12	37	193
40	147	221	368	134	204	246	23	57	314
50	241	361	602	218	333	401	41	90	511

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	9	30	9	14	16	−9	−5	14
10	1	13	47	14	21	25	−13	−8	22
20	2	18	74	22	33	40	−20	−15	35
30	4	27	119	34	53	64	−30	−26	56
40	9	41	192	55	85	102	−46	−44	90
50	18	62	310	89	137	165	−71	−75	146

Interpretation At first sight the development of the economy appears healthy. Of course, the saved wealth of all household groups far exceeds the gross domestic product, that is, the value of all goods and service in the economy. In addition there is a significant transfer of interest from the two lower income groups to the highest. This increases the relative differences among the wealth development of households.

2. Scenario: Interest rate constant 3%**, economic growth initially** 5% **then decreasing to** 1.12% **after** 50 **years.** Strong economic growth at the start decreases constantly over time and falls after 17 years to below 3%, the level of the interest rate.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	3	5.0	1.05	1	310	1045	20	39	80
10	3	3.7	1.59	18	432	1622	31	58	122
20	3	2.74	2.17	19	582	2491	37	75	175
30	3	2.03	2.74	−57	722	3752	36	88	238
40	3	1.51	3.25	−297	791	5536	25	91	315
50	3	1.12	3.69	−807	691	8010	0	81	407

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1	10	45
10	30	45	76	29	44	53	2	14	69
20	37	58	103	39	60	72	−2	15	103
30	37	66	130	50	76	91	−13	12	147
40	32	67	155	59	90	108	−34	1	206
50	22	59	176	67	102	123	−67	−21	284

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	9	30	9	14	16	−9	−5	14
10	0	13	47	14	21	25	−13	−8	21
20	1	17	72	20	31	38	−20	−14	34
30	−1	21	108	29	45	54	−31	−23	54
40	−8	24	160	40	61	74	−48	−38	86
50	−22	21	232	53	81	97	−75	−59	134

Interpretation The wealth of the lowest income household group begins to fall after 17 years. Although the economy still continues to grow after 50 years, the wealth of even the middle household group falls after the 40^{th} year. In contrast, the wealth of the highest income group rises all the more strongly. The gap between the incomes of the different household groups is considerably greater than in the previous 1^{st} scenario, and in the long term the economy becomes unstable.

3. Scenario: Interest rate constant 3%, economic growth constant at 4%. From the initial data this situation is similar to the 1^{st} scenario. The difference is that here the rate of economic growth is 1% less, though at a constant 4% it is 1% higher than the interest rate.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	3	4	1.04	1	310	1045	20	39	80
10	3	4	1.54	11	427	1617	29	56	120
20	3	4	2.28	19	583	2488	42	81	181
30	3	4	3.37	24	789	3810	62	117	272
40	3	4	4.99	19	1062	5810	90	170	407
50	3	4	7.39	−2	1421	8829	132	247	609

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1.0	10	45
10	29	44	74	28	43	52	1	13	69
20	42	64	110	42	63	77	1	18	104
30	61	94	162	62	94	113	0	23	158
40	90	139	240	91	139	168	−1	30	239
50	132	205	355	135	206	249	−3	41	361

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	9	30	9	14	16	−9	−5	14
10	0	12	46	14	21	25	−13	−8	21
20	1	17	71	20	31	37	−20	−14	34
30	1	23	109	30	47	56	−30	−24	53
40	1	31	167	45	69	84	−45	−38	83
50	0	41	254	68	103	125	−68	−62	129

Interpretation Despite constant economic growth at 4%, after 50 years the wealth of the lowest income household group finally falls into debt. The economy becomes unstable.

4. Scenario: Interest rate constant 3%**, economic growth initially** 4%**, decreasing to** 0.2%**.** The situation is similar to the 2^{nd} scenario. Here, however, the initial growth at 4% is 1% less, and after 50 years growth stagnates.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	3	4.0	1.04	1	310	1045	20	39	80
10	3	2.2	1.39	−2	417	1608	23	49	113
20	3	1.2	1.63	−73	506	2412	18	52	148
30	3	0.66	1.77	−278	521	2412	4	46	187
40	3	0.36	1.86	−674	386	5038	−15	29	235
50	3	0.2	1.91	−1291	−12	7072	−37	1	291

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1	10	45
10	23	37	67	25	39	47	−2	11	66
20	19	37	78	30	45	55	−12	7	93
30	12	30	85	32	49	60	−28	−4	127
40	3	16	89	34	52	63	−49	−23	173
50	0	0	85	35	53	64	−72	−52	227

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	9	30	9	14	16	−9	−5	14
10	0	12	46	13	20	25	−13	−8	22
20	−2	15	70	19	29	35	−21	−14	35
30	−8	16	102	25	38	46	−33	−23	55
40	−19	12	146	32	49	59	−51	−36	87
50	−37	1	205	39	59	72	−76	−58	134

Interpretation For the two lower earning household groups the economic situation becomes dramatic. For both groups income falls below consumption expenditure, saved wealth is eventually used up, and debt occurs. The incomes of the highest income group also begin to fall after 47 years, though this is greatly outweighed by the growth in income. The economy overall becomes unstable.

5. Scenario: Interest rate constant 0%**, economic growth initially** 4%**,decreasing to** 0.2%**.** The growth data are the same as in the 4^{th} scenario, but now the interest rate is set at zero.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	0	4.0	1.04	1	301	1015	20	30	50
10	0	2.2	1.39	13	313	1194	27	40	67
20	0	1.2	1.63	28	328	1414	31	47	78
30	0	0.66	1.77	44	344	1661	34	51	85
40	0	0.36	1.86	62	362	1924	36	54	89
50	0	0.2	1.91	80	380	2197	37	55	92

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1	1	15
10	27	40	67	25	39	47	1	1	20
20	31	47	78	30	45	55	2	2	23
30	34	51	85	32	49	60	2	2	26
40	36	54	89	34	52	63	2	2	27
50	37	55	92	35	53	64	2	2	28

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0

Interpretation In contrast to the results of the 4^{th} scenario, here despite falling economic growth the economy remains stable.

6. Scenario: Interest rate constant 0%, economic growth initially 2%, decreasing to 0%. In this scenario the initial economic growth is reduced from 4% to 2% compared to the previous scenario, falling further towards zero. The interest rate remains at zero.

Results of the dynamic analysis:

Years	**Growth**			**Wealth**			**GDP**		
	r	y	Σ	G1	G2	G3	G1	G2	G3
0	0	2.0	1.02	1	301	1015	20	30	50
10	0	0.6	1.14	12	312	1176	22	33	56
20	0	0.18	1.18	23	323	1347	23	35	58
30	0	0.05	1.19	35	335	1521	23	35	58
40	0	0.02	1.19	46	346	1696	23	35	58
50	0	0.0	1.19	58	358	1871	23	35	58

Years	**Income**			**Consumption**			**Saving**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	20	30	50	19	29	35	1	1	15
10	22	33	56	21	32	39	1	1	17
20	23	35	58	22	33	40	1	1	17
30	23	35	58	22	34	41	1	1	17
40	23	35	58	22	34	41	1	1	18
50	23	35	58	22	34	41	1	1	18

Years	**Interest income**			**Int. consumption**			**Interest transfer**		
	G1	G2	G3	G1	G2	G3	G1	G2	G3
0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0

Interpretation Even in the case where economic growth is initially slow and finally stagnates, the economy remains stable in the long term.

4 Model including the State

Extending the model to include the economic actor State is treated in detail in another article. Qualitatively, however, the central macroeconomic properties of the state can be derived from the information given so far.

1. Rising interest costs and levelling off or stagnating economic growth lead to a reduction in wages. This in turn leads to a reduction in the purchasing power of the workforce, which finally leads to lower income for firms. The state can act against this cycle by taking on a part of the debt burden. In this way it can itself act as a firm and promote employment, wages and consumption potential
2. Through taxes and subsidies the state can carry out a redistribution of wealth and thereby favour individual household groups.

Interventions by the state can lessen for a while the serious negative symptoms of the economic crisis caused by the payment of interest. It cannot alter the causes of the crisis. In the long term state activities end in state bankruptcy, which for example can lead to currency reform.

5 Summary and conclusion

In the economy studied here the following correspondences hold:

Households	Balance equation	$S = W + R - C$
	Wealth	$V = V + S$
	Interest yields	$R = rV$
Firms	Balance equation	$I = W + R - P$
	Precondition for $S = I$	$Rf = Rh =: R$
	Gross domestic product	$Y = P + I = C + S = W + R$

The decomposition

$$Y = \frac{R}{Y}Y + \frac{W}{Y}Y$$

is interpreted as the division of gross domestic product into an interest part $\frac{R}{Y}$ and a wage part $\frac{W}{Y}$.

Interest yields per household group are

$$R_t^i = r_t V_{t-1}^i$$

while interest payments via consumption of the i-th group have the value

$$\rho_t^i = \frac{C_t^i}{C_t} R_t.$$

Dynamic Analysis complements comparative-static analysis, and tries to shed light on the long term behaviour of economic systems and the long term effect of macroeconomic influencing factors. In the model presented here, interest and economic growth are exogenous factors whose influence on macroeconomic factors such as incomes, consumption, saving and wealth is investigated.

From the results of the analysis we can draw the following conclusions:

1. *The aggregated saving V in an economy is exactly equal to the aggregated debts. The payment of interest on saving capital and the increase in saving accumulation that this causes lead to a parallel rise in indebtedness and corresponding interest payments by firms. ($S = I$, equation (6)).*

2. *If gross domestic product grows less strongly than the interest part, then incomes in the economy shrink. In this case a larger part of the gross national product flows to the owners of capital via interest payments, while the wages portion falls by a matching amount. That is why efforts to limit the interest portion of gross domestic product lead to constant economic growth. ($Y = W + R$, equation (7)).*

3. *If the interest portion of gross domestic product grows, households with low wage incomes can afford increasingly less consumption, which can be characterised as impoverishment. The impoverishment of an increasingly large percentage of household groups ultimately destabilises the economy. (Consequence of 2.).*

4. *Groups with relatively low wealth receive little or no interest income, but pay so much interest through their consumption that they are net interest payers. For groups of relatively wealthy households, in contrast, interest income exceeds interest payments. This group consists of net interest receivers. Hence a steady stream of interest flows in the economy from net interest payers to net interest receivers (Interest balance: $R_t^i - \rho_t^i = rV_{t-1}^i - \frac{C_t^i}{C_t} R_t$, equation (14)).*

5. *The development of income, wealth and consumption over time remains stable for every household group if the interest rate in the economy is set at zero*[1]*. This is particularly true when economic growth falls or stagnates.*

Each of these claims is confirmed by the results of the simulation. Assuming that the models presented here are so comprehensive that the results of dynamic analysis can be applied to real economies, then we can draw the following conclusion: the payment of interest on capital over the long term only lacks destabilising economic effects if the economy grows constantly and unlimited in time, i.e. exponentially. In view of the finiteness of the earth's resources, constant economic growth is neither desirable nor possible. An economic order

[1] Nevertheless, it seems to be conceptually problematic that savings, which are future claims on the services and products of an economy, can rise to such an extent that they amount to or even exceed GDP.

which wishes to remain stable over time must therefore reject the concept of paying interest on wealth.

There may be a need for improvement in our economies. But if the model proposed in this article does reflect essential features of reality, then no economic reform can be successful unless the problems caused by the payment of interest on wealth in the first place are recognised and solved.

References

[1] Felderer, B., Homburg, S. (2005), Makroökonomik und neue Makroökonomik, 9. Auflage, Springer.

[2] Mankiw, N.G. (2003), Makroökonomik, 5. Auflage, Schäffer-Poeschel.

[3] Otto, M. (2004), Makroökonomik, WRW-Verlag.

[4] Stiglitz, J.E. (1999), Volkswirtschaftslehre, 2. Auflage, Oldenbourg.